For each exposure we can calibrate both astrometry and photometry using measurements of objects matched with reference catalog. But the number of objects are limited by reference catalog and the we may be affected by some systematic effects. Using multiple exposures and simultaneous fitting on them enable us more reliable and systematics free calibration. This is known as "uber-calibration" and applied to SDSS and Pan-STARRS1 photometry. In the pipeline we apply the same kind of method to each tract. "Uber-calibration" for entire survey region is a separate problem and not yet implemented.

In our meas_mosaic package we use both catalog matched objects and non-matched objects. Non-matched objects will be spatially matched between exposures based on astrometry determined by chip based analysis. Requirements for non-matched objects having consistent positions/fluxes contribute to better relative (internal) consistency and requirements for catalog matched objects contribute to absolute (external) consistency.

In astrometry optical distortion of the system is modeled by polynomial functions of focal plane coordinates. We determine the coefficients of polynomials for each exposure. The locations of CCDs on the focal plane (offsets and rotations) are also fitting parameters. In photometry correction for the flat-field is modeled by polynomial functions of focal plane coordinates. Same as astrometry, the coefficients of polynomials are determined. Relative flux scales between exposures are also determined.

The mathematical functional forms are the following.

In the following, super(sub)scription 's', 'e', and 'c' denotes stars, exposures and chips, respectively. (x, y) is a pixel coordinate within a chip, (u, v) is a focal plane coordinate, (ξ, η) is an projected celestial coordinate of the sky at (A, D).

For astrometry,

I'm solving the equation for Δa_k^e, Δb_k^e, ΔX_c, ΔY_c, and $\Delta\theta_c$ until it converges. For non-matched sources $\Delta\alpha^s$ and $\Delta\delta^s$ are also solved. For matched sources their catalog values are used for their (α, δ) and they are fixed. χ^2 equation is linearized at eq (5). Initial values are estimated from the fit to matched sources for individual exposures. Only multiply $(n \geq 2)$ observed non-matched sources are used in the fitting.

$$\begin{pmatrix} u^{s,e} \\ v^{s,e} \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} x^{s,e,c} \\ y^{s,e,c} \end{pmatrix} + \begin{pmatrix} X_c \\ Y_c \end{pmatrix} = \begin{pmatrix} x^{s,e,c}\cos\theta_c - y^{s,e}\sin\theta_c + X_c \\ x^{s,e,c}\sin\theta_c + y^{s,e}\cos\theta_c + Y_c \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} \xi^{s,e} \\ \eta^{s,e} \end{pmatrix} = \begin{pmatrix} \xi(\alpha^s, \delta^s, A^e, D^e) \\ \eta(\alpha^s, \delta^s, A^e, D^e) \end{pmatrix} \quad (2)$$

$$\begin{aligned}
\chi^2 &= \sum_e \sum_s \left\{ \xi^{s,e} - \sum_k a_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} \right\}^2 + \sum \left\{ \eta^{s,e} - \sum_k b_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} \right\}^2 && (3) \\
&= \sum_e \sum_s \left\{ \xi^{s,e} - \sum_k a_k^e [x^{s,e}\cos\theta_c - y^{s,c}\sin\theta_c + X_c]^{i(k)} [x^{s,e}\sin\theta_c + y^{s,c}\cos\theta_c + Y_c]^{j(k)} \right\}^2 \\
&+ \sum_e \sum_s \left\{ \eta^{s,e} - \sum_k b_k^e [x^{s,e}\cos\theta_c - y^{s,c}\sin\theta_c + X_c]^{i(k)} [x^{s,e}\sin\theta_c + y^{s,c}\cos\theta_c + Y_c]^{j(k)} \right\}^2 && (4) \\
&= \sum_e \sum_s \Bigg\{ \xi^{s,e} + \frac{\partial \xi^{s,e}}{\partial \alpha^s}\Delta\alpha^s + \frac{\partial \xi^{s,e}}{\partial \delta^s}\Delta\delta^s \\
&\quad - \Bigg(\sum_k a_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} + \sum_k \Delta a_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} \\
&\quad + \sum_k a_k^e \cdot i(k) \cdot [u^{s,e}]^{i(k)-1} [v^{s,e}]^{j(k)} \Delta X_c + \sum_k a_k^e \cdot j(k) \cdot [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)-1} \Delta Y_c \\
&\quad + \sum_k \Bigg[a_k^e [u^{s,e}]^{i(k)-1} [v^{s,e}]^{j(k)-1} \\
&\quad \{ -i(k)[v^{s,e}](x^{s,e}\sin\theta_c + y^{s,e}\cos\theta_c) + j(k)[u^{s,e}](x^{s,e}\cos\theta_c - y^{s,e}\sin\theta_c) \} \Bigg] \Delta\theta_c \Bigg) \Bigg\}^2
\end{aligned}$$

$$
\begin{aligned}
+ \quad & \sum_e \sum_s \Bigg\{ \eta^{s,e} + \frac{\partial \eta^{s,e}}{\partial \alpha^s} \Delta\alpha^s + \frac{\partial \eta^{s,e}}{\partial \delta^s} \Delta\delta^s \\
& \quad - \Bigg(\sum_k b_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} + \sum_k \Delta b_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} \\
& \quad + \sum_k b_k^e \cdot i(k) \cdot [u^{s,e}]^{i(k)-1} [v^{s,e}]^{j(k)} \Delta X_c + \sum_k b_k^e \cdot j(k) \cdot [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)-1} \Delta Y_c \\
& \quad + \sum_k \Bigg[b_k^e [u^{s,e}]^{i(k)-1} [v^{s,e}]^{j(k)-1} \\
& \quad \{-i(k)[v^{s,e}](x^{s,e} \sin\theta_c + y^{s,e} \cos\theta_c) + j(k)[u^{s,e}](x^{s,e} \cos\theta_c - y^{s,e} \sin\theta_c)\} \Bigg] \Delta\theta_c \Bigg) \Bigg\}^2 \qquad (5) \\
= \quad & \sum_e \sum_s \left\{ A_x^{s,e} - \sum_k \Delta a_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} - B_x^{s,e} \Delta X_c - C_x^{s,e} \Delta Y_c - D_x^{s,e} \Delta\theta_c + \frac{\partial \xi^{s,e}}{\partial \alpha^s} \Delta\alpha^s + \frac{\partial \xi^{s,e}}{\partial \delta^s} \Delta\delta^s \right\}^2 \\
+ \quad & \sum_e \sum_s \left\{ A_y^{s,e} - \sum_k \Delta b_k^e [u^{s,e}]^{i(k)} [v^{s,e}]^{j(k)} - B_y^{s,e} \Delta X_c - C_y^{s,e} \Delta Y_c - D_y^{s,e} \Delta\theta_c + \frac{\partial \eta^{s,e}}{\partial \alpha^s} \Delta\alpha^s + \frac{\partial \eta^{s,e}}{\partial \delta^s} \Delta\delta^s \right\}^2 \qquad (6)
\end{aligned}
$$

$$A_x = \xi - \sum_k a_k [u]^{i(k)} [v]^{j(k)} \qquad (7)$$

$$A_y = \eta - \sum_k b_k [u]^{i(k)} [v]^{j(k)} \qquad (8)$$

$$B_x = \sum_k a_k \cdot i(k) \cdot [u]^{i(k)-1} [v]^{j(k)} \qquad (9)$$

$$B_y = \sum_k b_k \cdot i(k) \cdot [u]^{i(k)-1} [v]^{j(k)} \qquad (10)$$

$$C_x = \sum_k a_k \cdot j(k) \cdot [u]^{i(k)} [v]^{j(k)-1} \qquad (11)$$

$$C_y = \sum_k b_k \cdot j(k) \cdot [u]^{i(k)} [v]^{j(k)-1} \qquad (12)$$

$$D_x = \sum_k a_k [u]^{i(k)-1} [v]^{j(k)-1} \{-i(k)[v](x^s \sin\theta_c + y^s \cos\theta_c) + j(k)[u](x^s \cos\theta_c - y^s \sin\theta_c)\} \qquad (13)$$

$$D_y = \sum_k b_k [u]^{i(k)-1} [v]^{j(k)-1} \{-i(k)[v](x^s \sin\theta_c + y^s \cos\theta_c) + j(k)[u](x^s \cos\theta_c - y^s \sin\theta_c)\} \qquad (14)$$

For photometry,

We solve for $f_{i,j}$, dm^e, and m_0^s. m_0^s is the true magnitudes of stars and for catalog matched objects m_0^s will be replaced with m_{cat}^s and fixed to give absolute calibration.

$$\chi^2 = \sum_e \sum_s \left\{ m_0^s - \left(m^{s,e} + dm^e + \sum_{i+j \le n} f_{i,j} [u^{s,e}]^i [v^{s,e}]^j \right) \right\}^2 \qquad (15)$$